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Compiler Construction

Lecture 6: Top-down parsing and LL(1) parser construction 2020-01-24 Michael Engel



Overview

- Ambiguity of grammars revisited
- Elimination of left recursion
- Top-down parsing
 - Recursive descent parsers: structure and implementation
 - Table-driven LL(1) parsers
 - Table generation



2

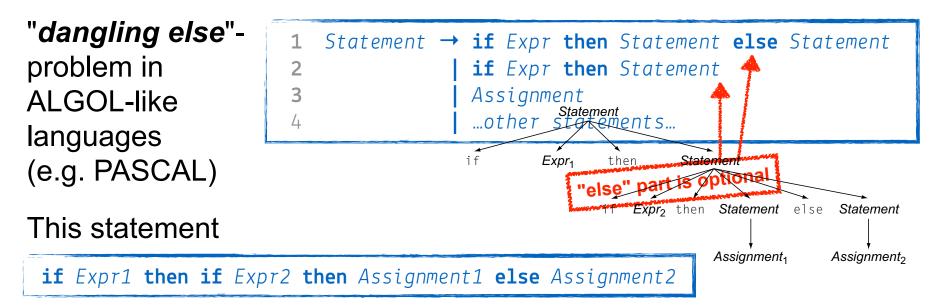
Ambiguity of grammars

3

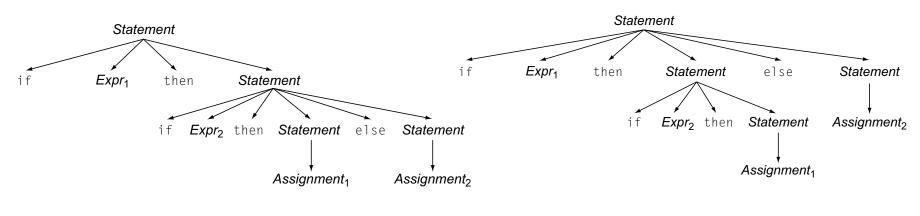
- For the compiler, it is important that each sentence in the language defined by a context-free grammar has a *unique* rightmost (or leftmost) *derivation*
- A grammar in which multiple rightmost (or leftmost) derivations exist for a sentence is called an *ambiguous grammar*
 - it can produce multiple derivations and multiple parse trees

 Multiple parse trees imply *multiple possible meanings for a* single program!

Ambiguity of grammars: example



has two distinct rightmost derivations with different behaviors:



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Syntax

analysis

4

Removing ambiguity



5

We can modify the grammar to include a rule that determined which **if** controls an **else**:

1	Statement	→	if Expr then	Statement	
2			<pre>if Expr then</pre>	WithElse else	Statement
3			Assignment		
4	WithElse	\rightarrow	<pre>if Expr then</pre>	WithElse else	WithElse
5			Assignment		

This solution restricts the set of statements that can occur in the **then** part of an **if-then-else** construct

- It accepts the same set of sentences as the original grammar
- but ensures that each else has an unambiguous match to a specific if



Removing ambiguity: example

The modified grammar has only one rightmost derivation for the example

1	Statement →	if Expr then Statement
2		<pre>if Expr then WithElse else Statement</pre>
3		Assignment
4	WithElse →	<pre>if Expr then WithElse else WithElse</pre>
5		Assignment
	and the second sec	

if Expr1 then if Expr2 then Assignment1 else Assignment2

Rule	Sentential form					
	Statement					
1	if Expr then Statement					
2	<pre>if Expr then if Expr then WithElse else Statement</pre>					
3	<pre>if Expr then if Expr then WithElse else Assignment</pre>					
5	if Expr then if Expr then Assignment else Assignment					

Syntax

Order of derivations

Rightmost:

rewrite, at each step, the rightmost nonterminal

Rule	Sentential form						
	Expr						
2	Expr Op name						
6	Expr × name						
1	"(" Expr ")" × name						
2	"(" Expr Op name ")" × name						
4	"(" Expr + name ")" × name						
3	"(" name + name ")" $ imes$ name						

$\begin{array}{cccc} 4 & Op & \rightarrow + \\ 5 & & | - \\ 6 & & | \times \\ 7 & & \vdots \end{array}$

1

2

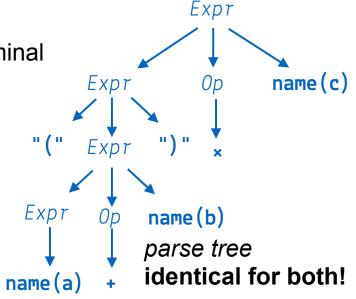
3

Leftmost: rewrite, at each step, the leftmost nonterminal

Rule	Sentential form					
	Ехрт					
2	Expr Op name					
1	"(" Expr ")" Op name					
2	"(" Expr Op name ")" Op name					
3	"(" name Op name ")" Op name					
4	"(" name + name ")" Op name					
6	"(" name + name ")" $ imes$ name					



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Expr → "(" Expr ")"

name

Expr Op name

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Syntax analysis

Left factoring

8

- Parsers (and scanners) only have a limited *lookahead* to upcoming tokens
- Example: given a production

```
A \rightarrow abcdef X gh \mid abcdef Y gh
```

the parser is unable to choose between the two production if it can only look one character ahead

- As with NFA→DFA conversion, if we can postpone the decision until it makes a difference, that works
 - Rewriting the grammar as
 - $A \rightarrow abcdef A'$
 - $A' \rightarrow X$ gh | Y gh

preserves the language by adding one production to collect a common prefix shared by several other productions



Left recursion

9

• Let's consider this grammar for a list of 'a's:

 $A \rightarrow Aa \mid a$

which derives the following words:

 $\begin{array}{cccc} A & \rightarrow & \mathbf{a} \\ A & \rightarrow & A\mathbf{a} & \rightarrow & \mathbf{a}\mathbf{a} \\ A & \rightarrow & A\mathbf{a} & \rightarrow & A\mathbf{a}\mathbf{a} & \rightarrow & \mathbf{a}\mathbf{a}\mathbf{a} \end{array}$

....

The production A → Aa is *left recursive*, the head (nonterminal symbol) always appears on the left side of the production

An equivalent grammar



• The same sequences can be generated by this grammar:



It derives the following words:

Eliminating left recursion

 If a nonterminal has *m* productions that are left recursive and *n* productions that are not

$$A \rightarrow A\alpha_{1} | A\alpha_{2} | A\alpha_{3} | \dots | A\alpha_{m}$$

$$A \rightarrow \beta_{1} | \beta_{2} | \beta_{3} | \dots | \beta_{n}$$

$$Greek letters (except ε) stand
for arbitrary combinations
of other (non-)terminals$$

we can introduce A' and rewrite the productions as (see [1]):

 $A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \beta_3 A' \mid \dots \mid \beta_n A'$

 $A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \alpha_3 A' \mid \dots \mid \alpha_m A' \mid \varepsilon$

- This generates the same language and removes (immediate) left recursion
 - "Immediate" because left recursion can also happen in several steps (indirectly), e.g. in the following productions

 $A \rightarrow B\mathbf{x}$ and $B \rightarrow A\mathbf{y}$ result in $A \rightarrow B\mathbf{x} \rightarrow A\mathbf{y}\mathbf{x}$

Here, A again shows up on the left when derived from A

Svnta

What can we do with CFGs now?



- So far, we have encountered (see also [2])
 - Context-Free Grammars, their derivations and syntax trees
 - Ambiguous grammars, and mentioned that there's no single, true way to disambiguate them (it depends on what we want them to stand for)
 - Left factoring, which always shortens the distance to the next nonterminal
 - Left recursion elimination, which always shifts a nonterminal to the right



Recursive descent parsing



13

- Example: grammar that models "if" and "while" statements:
- Let's make it a bit simpler:

```
P \rightarrow iCtSz \mid iCtSeSz \mid wCdSzC \rightarrow cS \rightarrow s
```

- Let us parse the string "ictsesz"
- A top-down parser begins at the *start symbol* P and chooses a production:

777

Recursive descent: what next?

 If we can only look ahead by one token and read an "i", we can choose between two productions:

```
P \rightarrow iCtSz| iCtSeSz
```

- · We cannot make this choice before seeing more of the token stream
- Left factoring makes this problem decidable with only one character of lookahead
- It generates the following grammar:

```
P \rightarrow iCtSP' \mid wCdSz
P' \rightarrow z \mid eSz
C \rightarrow c
S \rightarrow s
```

Syntax

Compiler Construction 06: Top-down, LL(1) parsing ¹⁵

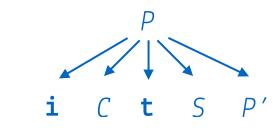
Recursive descent: what next?

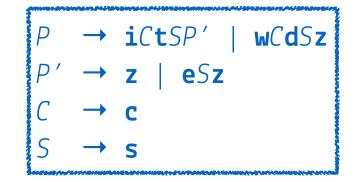
- Now we only have one production to choose from when reading an "i":
 - $P \rightarrow iCtSP'$

 and we can generate the *parse tree* equivalent to the derivation:

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Syntax analysis

Compiler Construction 06: Top-down, LL(1) parsing

Recursive descent: going down...

- Recursive descent implies that we follow the children of the current parse tree node down to the leaves (which must be terminal symbols)
- So let's see if we can parse "ictsesz"
- We follow the tree from *P* to its first child:

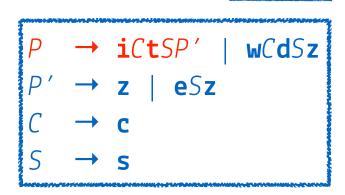
we have an "i" as lookahead
 ⇒ matches the first production for P!

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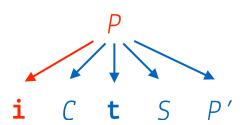
• Now, the remaining token stream is "ctsesz"

The input token sequence: ictsesz t the arrow indicates the parser's position in the token stream



Syntax

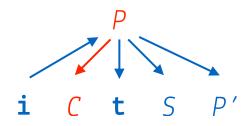
analvsis





Backtrack and repeat

- we have an "i" as lookahead \Rightarrow *match!*
- Now, the remaining token stream is "ctsesz"
 - We return (backtrack) to P to continue parsing:



The input token sequence: i ctsesz 1

iCtSP'

Syntax

analysis

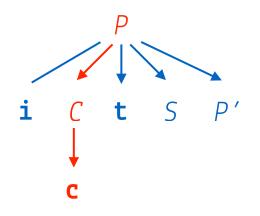
wCdSz

- This gives us the nonterminal C
- A nonterminal cannot match any token, so we need to pick another production



Pick the next production

- There is only one choice to expand C
 - When going from *P* to *C* in the previous step, we did not consume a token
- The lookahead is now c
 - Pick production C → c and expand the tree:





 \rightarrow **i**CtSP'

Syntax

analysis

wCdSz

The input token sequence: i ctsesz

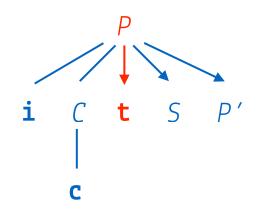
we have a "c" as lookahead ⇒ "tsesz"

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The next terminal symbol

- The next terminal symbol in P is t
- The lookahead is also t
 - Consume the token and expand the tree once more:





iCtSP'

Syntax

analysis

wCdSz

The input token sequence: ic tsesz

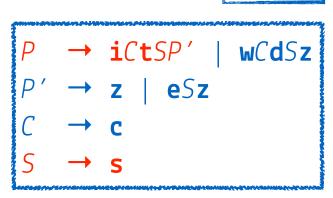
remaining token stream:"sesz"

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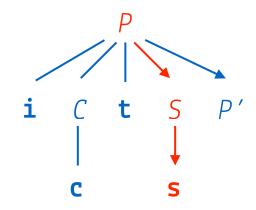
The next nonterminal symbol S

- The next nonterminal in the first production is 5, so we apply its production
- The lookahead is now s
 - This matches the pattern derived from
 - S, so we can expand the tree again:



Syntax

analysis



The input token sequence: ict sesz f

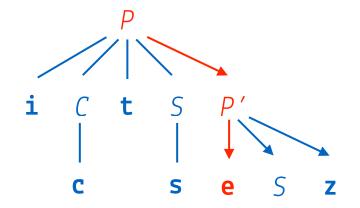
• remaining token stream: "esz"

The next nonterminal symbol S

- The final nonterminal in the first production is P'
- Now we have to choose between:

 $P' \rightarrow z \text{ and } P' \rightarrow eSz$

We can now choose the right production using only one token of lookahead!





iCtSP'

Syntax

analysis

wCdSz

The input token sequence: icts esz

remaining token stream: "sz"

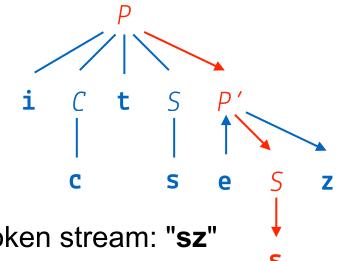
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The final steps

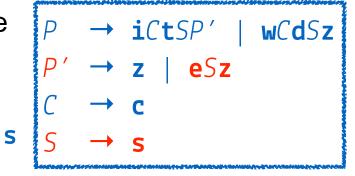
- The remaining steps are similar to ones we have already seen
- Take the next nonterminal symbol
 - S and match the input to production S

We can again choose the right production using only one symbol of lookahead!





22



The input token sequence: ictse sz

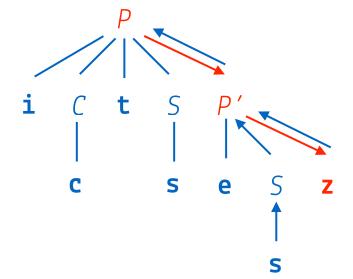
remaining token stream: "sz"

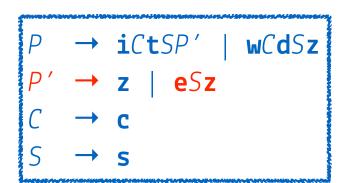
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Validated!

- The remaining nonterminal in the production P'→eSz is z
- This matches the remaining input token
 - → we backtrack and find no further children
 - \rightarrow we we able to match all characters, thus the input matches our grammar





Syntax

analysis

The input token sequence: ictses z → ictsesz ↑ ↑ ✓

23



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Compiler Construction 06: Top-down, LL(1) parsing

Top-down parsing summarized

- Predictive parsing by recursive descent:
 - Start from the start symbol (top)
 - Verify terminals
 - Pick a unique production for nonterminals based on the lookahead

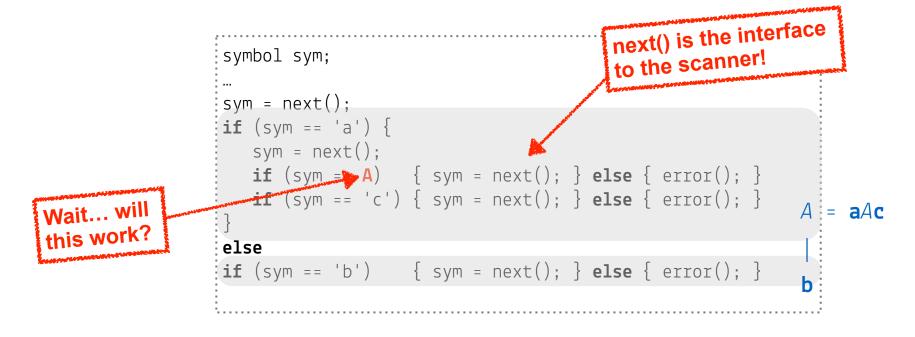


Syntax

- Expand the syntax tree by productions and recursively treat the new subtree in the same way
- This requires that the grammar is suitable, but we can adapt them somewhat
 - Left factor where a common lookahead prevents picking the right production
 - Eliminate left-recursive productions
 - We only saw left factoring in action so far, but let's do one other grammar

Implementing recursive descent

- Recursive descent parsers can easily be implemented by hand
- Example: parsing A = aAc | b
- We can naively try to implement the parser like this:



Syntax

Correct implementation

- Example: parsing A = aAc | b
- Whenever we encounter a *nonterminal* such as A we have to parse its *production*!
 Let us implement the
- Let us implement the parser as a function:

A() from main to ensure the parser works correctly in all cases? symbol sym; Recursively calling the parser for A allows void A(void) { to parse arbitrarily nested inputs! **if** (sym == 'a') { sym = next(); A(): **if** (sym == 'c') { sym = next(); } **else** { error(); } = aAcelse { sym = next(); } else { error(); } **if** (sym == 'b')

Some more implementation hints (not in C) can be found in [3]

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Compiler Construction 06: Top-down, LL(1) parsing ²⁶

Syntax

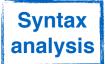
Table-driven parsing

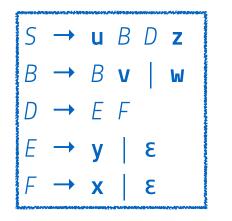


- As with scanners, coding a recursive descent parser for a complex language is lots of work and error prone
- Idea: use tables to configure the parser
 - parser makes decisions based on indexing (nonterminal, terminal) pairs and find a single production
- To make that table, it's a good idea to determine
 - What can the strings derived from a nonterminal begin with?
 - Which nonterminals can vanish, so that the lookahead symbol is actually part of the *next* production to choose?
 - What can come directly after a nonterminal that can vanish? (where 'vanish' means that there is a production $X \rightarrow \epsilon$, so that nonterminal X disappears from the intermediate form in the derivation without consuming any characters from the input token stream)



Another example grammar



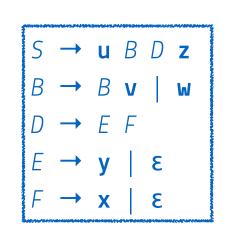


It doesn't model anything in particular, it's just a useful example



FIRST

- The set FIRST(α) is the set of terminals that can appear to the left in α
 - α is any combination of terminals and nonterminals



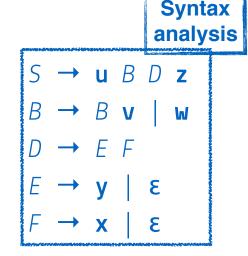


- If we tabulate FIRST for all the heads in the grammar, we obtain
 - FIRST(5) = {u} u begins the only production
 - FIRST(B) = {w} however many times B→Bv is taken,
 w appears on the left in the end
 - FIRST(E) = {y} only production that derives any terminal
 - $FIRST(F) = {x} ditto$
 - FIRST(*D*) = {y,x}
 - y because $D \rightarrow EF \rightarrow yF$
 - **x** because $D \rightarrow EF \rightarrow F \rightarrow \mathbf{x}$ (*E* can disappear by $E \rightarrow \epsilon$)



FOLLOW

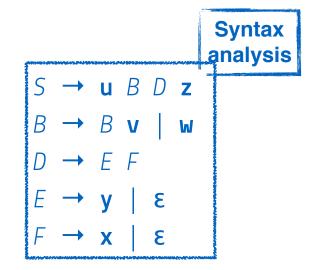
- FOLLOW (*N*) for a nonterminal *N* is the set of terminals that can appear directly to its right
 - In order to find these, you have to examine all the places N appears in production bodies, and find the terminals directly to its right



- If it has a nonterminal on its right, you have to follow all its productions too, and find out what can come up instead of it
 - That will be its FIRST set
- If it has a nonterminal that can vanish to its right, you have to look at what comes afterwards...
 - ...and in general, collect all the terminals that can appear to the right in one way or another
- This is a little trickier than FIRST, but it can be done manually
 - See fig. 3.8, p. 106 in [4] for an algorithm to compute FOLLOW

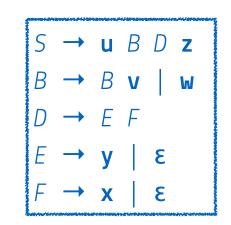
FOLLOW for our grammar

- FOLLOW(5) = {\$} (the end of input)
- FOLLOW(B) = {v, x, y, z} taken from the derivations
 - $\cdot S \rightarrow uBDz \rightarrow uBvDz$
 - $\cdot S \rightarrow uBDz \rightarrow uBEFz \rightarrow uBFz \rightarrow uBxz$
 - $\cdot S \rightarrow uBDz \rightarrow uBEFz \rightarrow uByFz$
 - $S \rightarrow uBDz \rightarrow uBEFz \rightarrow uBFz \rightarrow uBz$
- FOLLOW(D) = {z} (from $S \rightarrow uBDz$)
- FOLLOW(E) = {x, z} taken from the derivations
 - $\cdot S \rightarrow uBDz \rightarrow uBEFz \rightarrow uBExz$
 - $S \rightarrow uBDz \rightarrow uBEFz \rightarrow uBEz$
- FOLLOW(F) = $\{z\}$ from $S \rightarrow uBDz \rightarrow uBEFz$



Nullability

- A nonterminal is *nullable* if it can produce the empty string (in any number of steps)
 - Here, the notation might be different between various textbooks





- E.g., the Aho/Ullman/Seti/Lam "Dragon book" [5] (one of the standard compiler textbooks) denotes this by putting ε in the FIRST set
- We denote it by keeping a separate record
- To summarize,
 - nullable (5) = no there are terminals in the only production
 - nullable (B) = no there are terminals in both productions
 - nullable (E) = yes it produces $E \rightarrow \epsilon$
 - nullable (F) = yes it produces $F \rightarrow \epsilon$
 - nullable (D) = yes $-D \rightarrow EF \rightarrow F \rightarrow \epsilon$

Building the parsing table

- Obtain the FIRST and FOLLOW sets and nullable information for your grammar
- Consider every production X→ α in the grammar, and apply two rules
 - Enter the production $X \rightarrow \alpha$ at (X, t) where t is in FIRST (α)
 - When α→*ε, enter the production X→α at (X,t) where t is in FOLLOW(X)

Oops, a left recursion!

This will not work, expanding B on lookahead 'w' requires a choice the parser cannot make

	u	W	v	X	у	Z
S	S→uBDz					
В		B→w B→Bv				
D				D→EF	D→EF	
Е					E→y	
F				F→x		

Syntax

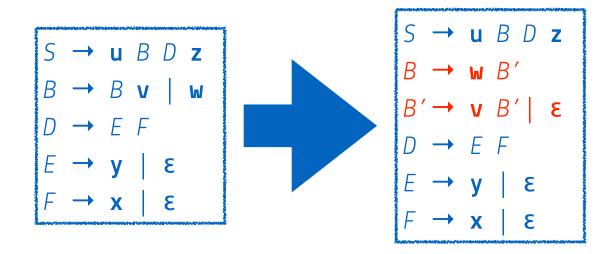
Fix the grammar



• Eliminating left recursion gives us

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- Update the FIRST, FOLLOW, nullable sets after the change:
 - $FIRST(B) = \{w\}, FOLLOW(B) = \{x, y, z\}, nullable(B) = no$
 - $FIRST(B') = \{v\}$, $FOLLOW(B') = \{x, y, z\}$, nullable(B') = yes

This is better... after rule 1



	u	W	V	x	у	z
5	S →u BD z					
В		B→wB′				
B′			B′ →v B′			
D				D→EF	D→EF	
Е					E→y	
F				F→x		



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Now apply rule 2



Where nonterminal symbols are *nullable*, insert at FOLLOW

	u	W	V	x	у	z
5	S→uBDz					
В		B→wB′				
B'			B′ →v B′	B′ →ε	<i>B′</i> → ε	B′ →ε
D				D→EF	D→EF	D→EF
Е				Ε→ε	E→y	E→ε
F				F→x		F→ε



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Result: a LL(1) parse table

38

- There is only one rule to choose from given a combination (NT, T) of a nonterminal and a terminal symbol
- Thus, the parse tree can be built deterministically by following the method from the first example
 - Pick productions for NTs by looking them up in the table
 - Encountering a combination without production \Rightarrow error
- The LL(1) parse table can, of course, also be constructed by an algorithm that processes (parses) the input grammar
 - See [4], fig. 3.12, p. 113 (note: the book adds the set FIRST⁺ to simplify notation)
- This is the first step to create a *parser generator* (also called *compiler compiler*)

So far, so good...

39

- Most programming language constructs can be expressed in a backtrack-free grammar
- Predictive parsers for these are simple, compact, and efficient
 - They can be implemented in a number of ways, including handcoded, recursive descent parsers and generated LL(1) parsers, either table driven or direct coded
- The primary drawback of top-down, predictive parsers lies in their inability to handle left recursion
 - Left-recursive grammars model the left-to-right associativity of expression operators in a more natural way than right-recursive grammars
- What lies ahead?
 - More parsing: bottom up -LR(1) parsers
 - These are the basis for many parser generators, e.g. yacc/bison

References

- [1] A.V. Aho, S.C. Johnson, J.D. Ullman: Deterministic parsing of ambiguous grammars Communications of the ACM, August 1975, doi:10.1145/360933.360969
- [2] D.J. Rosenkrantz, R.E. Stearns:

Properties of Deterministic Top Down Grammars

Information and Control. 17 (3): 226–256, 1970. doi:10.1016/s0019-9958(70)90446-8

[3] Niklaus Wirth:

Compiler Construction

Original version: Addison-Wesley 1996, ISBN 0-201-40353-6 Revised edition 2017 freely available at https://inf.ethz.ch/personal/wirth/CompilerConstruction/index.html

 in this small book of a bit more than 100 pages, Wirth explains the design and implementation of a small compiler for a subset of the Oberon language. This book is rather implementation-oriented, so don't expect too much theoretical detail

[4] Keith Cooper and Linda Torczon:

Engineering a Compiler (second Edition) ISBN 9780120884780 (hardcover), 9780080916613 (ebook)

[5] Alfred Aho, Monica S. Lam, Ravi Sethi, Jeffrey Ullman: Compilers: Principles, Techniques, and Tools (second edition) Addison-Wesley 2006, ISBN 978-0321486813



Syntax analysis